Directions:
- **Examples** are demonstrated by TA. You should watch the TA working through the problem and takes notes.
- **Exercises** are for you to work on with/without the help of TA. You will be graded on your work for the exercises. Always show your work!
- Each part is worth 1 point. There are 10 parts in total.

Example 1:

A jury of 6 people was selected from a group of 20 potential jurors, of whom 8 were African American and 12 were white. The jury was supposedly randomly selected, but it contained only 1 African American member. Do you have any reason to doubt the randomness of the selection?

Exercise 1:

It's Halloween! Luca goes around his neighborhood for trick or treat. At Dave's house, there are 23 candies in the basket: 10 Reese's, 7 Kit Kats and 6 Milky Ways. Dave lets each kid pick 3 candies out of the basket without looking!
Luca likes peanut butter so he's secretly hoping for a Reese's!

a)   What is the probability that Luca will get at least 2 Reese's?

b)   What is the probability that Luca will get none of the Reese's?

Example 2:

Suppose the number of air bubbles in window glass has Poisson distribution, with an average of 0.3 air bubbles per square foot. Find the probability of finding in a 4' by 3' window …

a)   … exactly 5 air bubbles.   b)   … at least 5 air bubbles.

Exercise 2:

The number of typos made by a student follows Poisson distribution with the rate of 1.5 typos per page. Assume that the numbers of typos on different pages are independent.

a)   Find the probability that there are at least 2 typos on a page.

b)   Find the probability that there are at most 10 typos in a 5-page paper.

c)   Find the probability that there is exactly one page with no typos in a 5-page paper.

Example 3:

If the moment-generating function of X is

$$M_X(t) = \frac{2}{5}e^t + \frac{1}{5}e^{2t} + \frac{2}{5}e^{3t},$$

Find the mean, variance, and pmf of X.

Exercise 3:

Suppose the moment-generating function of X is

$$M_X(t) = 0.2e^{-3t} + 0.45e^t + 0.35e^{3t}$$

a) Find $\mu = E(X)$

b)   Find $\sigma = \text{SD}(X)$.

Example 4:

Let X be a continuous random variable with the probability density function

$$f(x) = \frac{C}{x^4}, \qquad x > 5, \qquad\qquad \text{zero otherwise.}$$

a)   Find the value of C that would make $f(x)$ a valid probability density function.

b)   Find the 80th percentile of the distribution of X, $\pi_{0.80}$.

c)   Find the expected value of X, $E(X)$.

Let X be a continuous random variable with the probability density function

$$f(x) = C\frac{x^3}{2}, \qquad 2 \le x \le 4, \qquad\qquad \text{zero otherwise.}$$

a)   Find the value of C that would make $f(x)$ a valid probability density function.

b)   Find the expected value of X, $E(X)$.

c)   Find the median of the probability distribution of X.