STAT 400
Summer 2021

Discussion - Lab 04

Instructor: Ha Nguyen
Student:

Directions:

- **Examples** are demonstrated by TA. You should watch the TA working through the problem and takes notes.
- **Exercises** are for you to work on with/without the help of TA. You will be graded on your work for the exercises. Always show your work!
- Each part is worth 1 point. There are 10 parts in total.

Example 1:

Let X and Y have the joint p.d.f.

$$f_{X,Y}(x, y) = Cx^2 y, \quad 0 < x < 4, \quad 0 < y < \sqrt{x}, \qquad \text{zero elsewhere.}$$

a) Find the value of C so that $f_{X,Y}(x, y)$ is a valid joint p.d.f.

b) Find the marginal probability density function of X, $f_X(x)$.

c) Find the marginal probability density function of Y, $f_Y(y)$.

d) Are X and Y independent? If not, find $\text{Cov}(X, Y)$.

Exercise 1:

Let the joint probability density function for (X, Y) be

$$f(x,y) = x + y, \quad x > 0, \quad y > 0, \quad x + 2y < 2, \qquad \text{zero otherwise.}$$

a) Find the marginal p.d.f. of X, $f_X(x)$.

b) Find the marginal p.d.f. of Y, $f_{\mathrm{Y}}(y)$.

c) Find the probability $\mathrm{P}(\mathrm{Y} > \mathrm{X})$.

d) Are X and Y independent? If not, find $\mathrm{Cov}(\mathrm{X}, \mathrm{Y})$.

Example 2:

Let X and Y be two independent Exponential random variables with mean θ and θ, respectively. Let $W = X + Y$. What is the probability distribution of W?

Exercise 2:

Let X and Y be two independent Binomial random variables with the number of trials are n_1 and n_2, respectively, and the probability of success p (same for both X and Y).

Let $W = X + Y$.

You **must** justify your answer like Example 4 – Lecture notes 5.2.

a) What is the probability distribution of W?

b) What is the conditional probability distribution of X given $W = k$?

Example 3:

Let $\theta > 0$ and let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with the probability density function

$$f_X(x) = f_X(x;\theta) = \frac{\theta}{2\sqrt{x}}\, e^{-\theta\sqrt{x}}, \qquad x > 0.$$

a) Find the method of moments estimator $\tilde{\theta}$ of θ.

b) Suppose $n = 4$, and $x_1 = 0.01$, $x_2 = 0.04$, $x_3 = 0.09$, $x_4 = 0.36$.

Find the method of moments estimate $\tilde{\theta}$ of θ.

c) Find the maximum likelihood estimator $\hat{\theta}$ of θ.

d) Suppose $n = 4$, and $x_1 = 0.01$, $x_2 = 0.04$, $x_3 = 0.09$, $x_4 = 0.36$.

Find the maximum likelihood estimate $\hat{\theta}$ of θ.

Exercise 3:

Let $X_1, X_2, \dots, X_n$ be a random sample of size *n* from a distribution with probability density function

$$f(x|\alpha) = \alpha^{-2} x e^{-x/\alpha}, \quad x > 0, \quad \alpha > 0$$

a) Obtain the maximum likelihood *estimator* of α, $\hat{\alpha}$.

b) Calculate the *estimate* when $x_1 = 0.35$, $x_2 = 3.20$, $x_3 = 1.75$, $x_4 = 2.5$.

c) Obtain the method of moments *estimator* of α, $\tilde{\alpha}$.

d) Calculate the *estimate* when $x_1 = 0.35$, $x_2 = 3.20$, $x_3 = 1.75$, $x_4 = 2.5$.