

**Due date:** 11:59 PM Tuesday 10/26/2021

*Directions:*

- **Examples** are demonstrated by TA. You should watch the TA working through the problem and takes notes.
- **Exercises** are for you to work on with/without the help of TA. You will be graded on your work for the exercises. Always show your work!
- Each part is worth 2 points. There are 5 parts in total.

**Example 1:**

Let the random variables  $X$  and  $Y$  have the joint pmf

$$f(x, y) = \frac{x + y}{32}, \quad x = 1, 2, \quad y = 1, 2, 3, 4.$$

a) Find the covariance of  $X$  and  $Y$ ,  $\text{Cov}(X, Y) = \sigma_{XY}$ .

b) Find the correlation coefficient of  $X$  and  $Y$ ,  $\text{Corr}(X, Y) = \rho_{XY}$ .

c) Find  $E(X | Y = 2)$ .

**Exercise 1:**

Let  $X$  and  $Y$  have the joint pmf

$$f(x, y) = \frac{x + 3y}{27}, \quad x = 1, 2, \quad y = 0, 1, 2$$

a) Find the covariance of  $X$  and  $Y$ ,  $\text{Cov}(X, Y) = \sigma_{XY}$ .

b) Find the correlation coefficient of  $X$  and  $Y$ ,  $\text{Corr}(X, Y) = \rho_{XY}$ .

c) Find  $E(X | Y = 2)$ .

**Example 2:**

Let  $X$  and  $Y$  be two independent Poisson random variables with mean  $\lambda_1$  and  $\lambda_2$ , respectively.  
Let  $W = X + Y$ .

a) What is the probability distribution of  $W$ ?

b) What is the conditional distribution of  $X$  given  $W = n$ ?

### Exercise 2:

Let  $X$  and  $Y$  be two independent Binomial random variables with the number of trials are  $n_1$  and  $n_2$ , respectively, and the probability of success  $p$  (same for both  $X$  and  $Y$ ).

Let  $W = X + Y$ .

You **must** justify your answer like Example 4 – Lecture notes 9.1 (Several Independent RVs).

a) What is the probability distribution of  $W$ ?

b) What is the conditional probability distribution of  $X$  given  $W = k$ ?