STAT 400
UIUC

1.3 – Conditional Probability

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The **conditional probability of A, given B** (the probability of event A, computed on the assumption that event B has happened) is

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \qquad (\text{ assuming } P(B) \neq 0).$$

Similarly, the **conditional probability of B, given A** is

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \qquad (\text{ assuming } P(A) \neq 0).$$

<u>Example 2</u>: (cont.)

The probability that a randomly selected student at Anytown College owns a bicycle is 0.55, the probability that a student owns a car is 0.30, and the probability that a student owns both is 0.10.

	C	C'	
B	0.10	0.45	0.55
B'	0.20	0.25	0.45
	0.30	0.70	1

$P(B) = 0.55$, $P(C) = 0.30$, $P(B \cap C) = 0.10$.

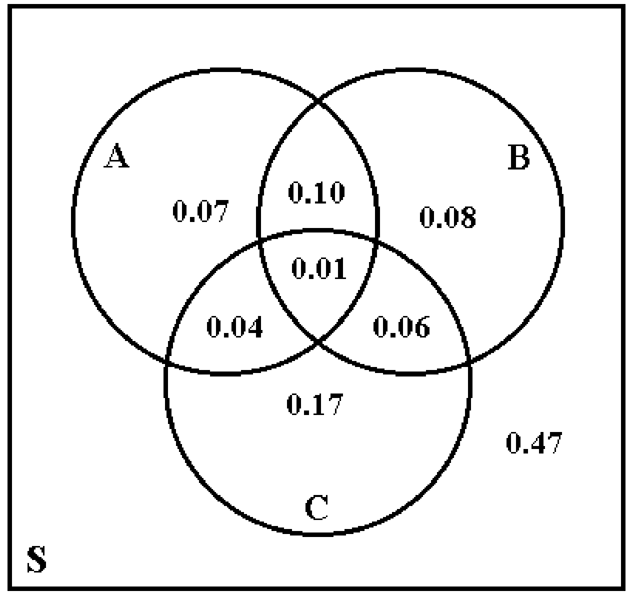
a) What is the probability that a student owns a bicycle, <u>given that he/she owns a car</u>?

b) <u>Suppose a student does not have a bicycle</u>. What is the probability that he/she has a car?

<u>Example 3</u>: (cont.)

Suppose

$P(A) = 0.22$,
$P(B) = 0.25$,
$P(C) = 0.28$,
$P(A \cap B) = 0.11$,
$P(A \cap C) = 0.05$,
$P(B \cap C) = 0.07$,
$P(A \cap B \cap C) = 0.01$.



Find the following:

a) $P(B \mid A)$

b) $P(B \mid C)$

c) $P(B \cap C \mid A)$

d) $P(B \cup C \mid A)$

e) $P(C \mid A \cup B)$

f) $P(C \mid A \cap B)$

g) $P(A \cap B \cap C \mid A \cup B \cup C)$

Multiplication Law of Probability.

If A and B are any two events, then

$$\mathbf{P(A \cap B) = P(A) \cdot P(B \mid A)}$$

$$\mathbf{P(A \cap B) = P(B) \cdot P(A \mid B)}$$

Example 7:

It is known that 30% of all the students at Anytown College live off campus. Suppose also that 48% of all the students are females. Of the female students, 25% live off campus.

a) What is the probability that a randomly selected student is a female and lives off campus?

b) What is the probability that a randomly selected student either is a female or lives off campus, or both?

c) What proportion of the off-campus students are females?

d) What proportion of the male students live off campus?

<u>Example 8</u>: Suppose that Joe's Discount Store has received a shipment of 25 television sets, 5 of which are defective. On the following day, 2 television sets are sold.

a) Find the probability that both of the television sets are defective.

b) Find the probability that at least one of the two television sets sold is defective.

<u>Example 9</u>: Cards are drawn one-by-one **without** replacement from a standard 52-card deck. What is the probability that …

a) … both the first and the second card drawn are ♥'s?

b) … the first two cards drawn are a ♥ and a ♣ (or a ♣ and a ♥) ?

c) … there are at least two ♥'s among the first three cards drawn?