

1.4 – Bayes’ Theorem

Review:

Law of Total Probability. $P(A) = P(A \cap B) + P(A \cap B')$

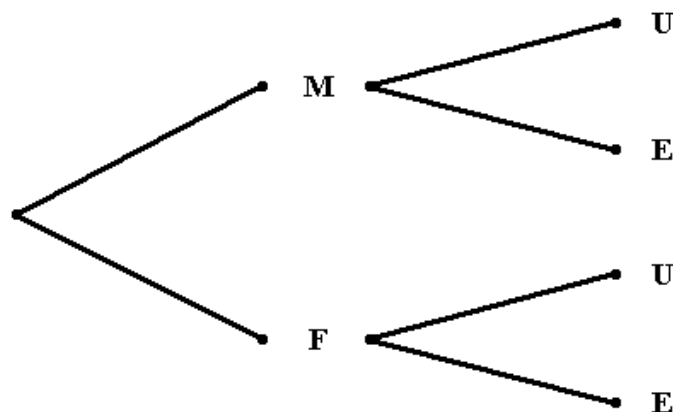
Example 1:

In Neverland, men constitute 60% of the labor force. The rates of unemployment are 6.0% and 4.5% among males and females, respectively. A person is selected at random from Neverland’s labor force.

a) What is the probability that the person selected is a male and is unemployed?

b) What is the probability that the person selected is a female and is unemployed?

	Unemployed	Employed	Total
Male			
Female			
Total			



c) What is the probability that the person selected is unemployed?

Law of Total Probability.

$$\begin{aligned} P(A) &= P(A \cap B) + P(A \cap B') \\ &= P(B) \cdot P(A | B) + P(B') \cdot P(A | B') \end{aligned}$$

In general,

$$P(A) = \sum_{i=1}^m P(B_i) \cdot P(A | B_i)$$

d) Suppose the person selected is unemployed. What is the probability that a male was selected?

Bayes' Theorem.

$$P(B | A) = \frac{P(B) \cdot P(A | B)}{P(B) \cdot P(A | B) + P(B') \cdot P(A | B')},$$

In general,

$$P(B_k | A) = \frac{P(B_k) \cdot P(A | B_k)}{\sum_{i=1}^m P(B_i) \cdot P(A | B_i)}, \quad k = 1, \dots, m.$$

Example 2:

In a presidential race in Neverland, the incumbent Democrat (D) is running against a field of four Republicans (R_1, R_2, R_3, R_4) seeking the nomination. Political pundits estimate that the probabilities of R_1, R_2, R_3 , and R_4 winning the nomination are 0.40, 0.30, 0.20, and 0.10, respectively. Furthermore, results from a variety of polls are suggesting that D would have a 55% chance of defeating R_1 in the general election, a 70% chance of defeating R_2 , a 60% chance of defeating R_3 , and an 80% chance of defeating R_4 . Assuming all these estimates to be accurate, what are the chances that D will be a two-term president?

Example 3:

In Anytown, 10% of the people leave their keys in the ignition of their cars. Anytown's police records indicate that 4.2% of the cars with keys left in the ignition are stolen. On the other hand, only 0.2% of the cars without keys left in the ignition are stolen. Suppose a car in Anytown is stolen. What is the probability that the keys were left in the ignition?

Example 4:

In a certain population, the proportion of individuals who have a particular disease is 0.025. A test for the disease is positive in 94% of the people who have the disease and in 4% of the people who do not.

a) Find the probability of receiving a positive reaction from this test.

b) If a person received a positive reaction from this test, what is the probability that he/she has the disease?

c) If a person received a negative reaction from this test, what is the probability that he/she doesn't have the disease?