

Events **A** and **B** are **independent** if and only if

$$P(B | A) = P(B) \quad P(A | B) = P(A)$$

$$P(A \cap B) = P(A) \cdot P(B)$$

Example 1:

The probability that a randomly selected student at Anytown College owns a bicycle is 0.55, the probability that a student owns a car is 0.30, and the probability that a student owns both is 0.10. Are events {a student owns a bicycle} and {a student owns a car} independent?

Events **A**, **B** and **C** are **independent** if and only if

$$P(A \cap B) = P(A) \cdot P(B),$$

and $P(A \cap C) = P(A) \cdot P(C),$

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Example 3:

Bart and Nelson talked Milhouse into throwing water balloons at Principal Skinner. Suppose that Bart hits his target with probability 0.80, Nelson misses 25% of the time, and Milhouse hits the target half the time. Assume that their attempts are independent of each other.

a) Find the probability that all of them will hit Principal Skinner.

b) Find the probability that exactly one of the boys will hit Principal Skinner.

c) Find the probability that at least one of the boys will hit Principal Skinner.

Idea: $P(\text{at least one of } A_i \text{ occurs}) = 1 - P(\text{none of } A_i \text{ occurs})$

$$P(A_1 \text{ or } A_2 \text{ or } \dots \text{ or } A_n) = 1 - P((\text{not } A_1) \text{ and } (\text{not } A_2) \text{ and } \dots \text{ and } (\text{not } A_n))$$

$$P(A_1 \cup A_2 \cup \dots \cup A_n) = 1 - P(A_1' \cap A_2' \cap \dots \cap A_n')$$

For independent events

$$P(A_1 \text{ or } A_2 \text{ or } \dots \text{ or } A_n) = 1 - P(\text{not } A_1) \cdot P(\text{not } A_2) \cdot \dots \cdot P(\text{not } A_n)$$

$$P(A_1 \cup A_2 \cup \dots \cup A_n) = 1 - P(A_1') \cdot P(A_2') \cdot \dots \cdot P(A_n')$$

Example 4:

Often On time Parcel Service (OOPS) delivers a package to the wrong address with probability 0.05 on any delivery. Suppose that each delivery is independent of all the others. There were 7 packages delivered on a particular day. What is the probability that at least one of them was delivered to the wrong address?

Example 5:

A major oil company has decided to drill independent test wells in the Alaskan wilderness. The probability of any well producing oil is 0.30. Find the probability that the fifth well is the first to produce oil.

Example 6:

An automobile salesman thinks that the probability of making a sale is 0.30. If he talks to five customers on a particular day, what is the probability that he will make exactly 2 sales? (Assume independence.)