

**Random Variables (Discrete)**  
**Expectation**  
**Mean, Variance, Standard Deviation**

A **random variable** associates a numerical value with each outcome of a random experiment.

A random variable is said to be **discrete** if it has either a finite number of values or infinitely many values that can be arranged in a sequence.

If a random variable represents some measurement on a continuous scale and therefore, capable of assuming all values in an interval, it is called a **continuous** random variable.

The **probability distribution** of a discrete random variable is a list of all its distinct numerical values along with their associated probabilities:

$x$	$f(x)$	
$x_1$	$f(x_1)$	
$x_2$	$f(x_2)$	!
$x_3$	$f(x_3)$	
⋮	⋮	
⋮	⋮	!
⋮	⋮	
$x_n$	$f(x_n)$	2) $\sum_{\text{all } x} f(x) = 1.$
	1.00	

Often a formula can be used in place of a detailed list.

**Example 1:**

A balanced (fair) coin is tossed twice.  
Let X denote the number of H's.  
Construct the probability distribution of X.

$x$	$f(x)$

**Example 2:**

Suppose a random variable X has the following probability distribution:

$x$	$f(x)$
10	0.20
11	0.40
12	0.30
13	0.10

a) Find the expected value of X,  $E(X)$ .

$x$	$f(x)$	$x \cdot f(x)$
10	0.2	
11	0.4	
12	0.3	
13	0.1	

$$E(X) = \mu_X = \sum_{\text{all } x} x \cdot f(x)$$

b) Find the variance of X,  $\text{Var}(X)$ .

$x$	$f(x)$	$(x - \mu_X)$	$(x - \mu_X)^2 \cdot f(x)$
10	0.2		
11	0.4		
12	0.3		
13	0.1		

$$\begin{aligned} \text{Var}(X) &= \sigma_X^2 = \sum_{\text{all } x} (x - \mu_X)^2 \cdot f(x) \\ &= E[(X - \mu_X)^2] \end{aligned}$$

$x$	$f(x)$	$x^2 \cdot f(x)$
10	0.2	
11	0.4	
12	0.3	
13	0.1	

$$\begin{aligned} \text{Var}(X) = \sigma_X^2 &= \sum_{\text{all } x} x^2 \cdot f(x) - [E(X)]^2 \\ &= E(X^2) - [E(X)]^2 \end{aligned}$$

c) Find the standard deviation of X, SD(X).

$$\text{SD}(X) = \sigma_X = \sqrt{\sigma_X^2}$$

d) Find the cumulative distribution function of X,  $F(x) = P(X \leq x)$ .

$x$	$f(x)$	$F(x)$
10	0.2	
11	0.4	
12	0.3	
13	0.1	

$$E(g(X)) = \sum_{\text{all } x} g(x) \cdot f(x)$$

$$E(a \cdot X + b) = a \cdot E(X) + b.$$

$$\text{Var}(a \cdot X + b) = a^2 \cdot \text{Var}(X).$$

$$\text{SD}(a \cdot X + b) = |a| \cdot \text{SD}(X).$$

**Example 3:**

Suppose  $E(X) = 7$ ,  $\text{SD}(X) = 3$ .

a)  $Y = 2X + 3$ . Find  $E(Y)$  and  $\text{SD}(Y)$ .

b)  $W = 5 - 2X$ . Find  $E(W)$  and  $\text{SD}(W)$ .

**Example 4:**

Suppose a discrete random variable  $X$  has the following probability distribution:

$$P(X = 0) = 2 - \sqrt{e}, \quad P(X = k) = \frac{1}{2^k \cdot k!}, \quad k = 1, 2, 3, \dots$$

a) Find  $E(X)$ .

b) Find  $\text{Var}(X)$ .

**Example 5:**

Let  $X$  be a discrete random variable with p.m.f.  $p(x)$  that is positive on the odd non-negative integers  $\{1, 3, 5, 7, 9, \dots\}$  and is zero elsewhere. Suppose

$$p(1) = c \text{ (unknown)}, \quad p(k) = \frac{1}{2^k}, \quad k = 3, 5, 7, 9, \dots$$

a) Find the value of  $c$  that makes this is a valid probability distribution.

Must have  $\sum_{\text{all } x} p(x) = 1.$

b) Find  $E(X)$ .

**Example 6:**

A grocer has shelf space for two units of a highly perishable item that must be disposed of at the end of the day if it is not sold. Each unit costs \$1.70 and sells for \$3.50. From past experience, the grocer knows that the demand for this item could be either 0, or 1, or 2 units. The grocer has to decide whether to stock one unit of this item or two units.

a) Construct the grocer's profit (payoff) in each of the following cases:

	demand = 0	demand = 1	demand = 2
stock 1 unit			
stock 2 units			

b) Suppose  $P(\text{demand} = 0) = 0.20$ ,  $P(\text{demand} = 1) = 0.30$ , and  $P(\text{demand} = 2) = 0.50$ . Find the expected payoff (profit) of both actions and choose the "optimal" action.

c) Suppose  $P(\text{demand} = 0) = 0.25$ ,  $P(\text{demand} = 1) = 0.30$ , and  $P(\text{demand} = 2) = 0.45$ . Find the expected payoff (profit) of both actions and choose the "optimal" action.

**Example 7:**

To save time and money when testing blood samples for the presence of a disease, the following procedure is sometimes used: Blood samples from a number of people are pooled together and the pooled sample is tested. If the pooled sample is negative, then no further testing needs to be done. If the pooled sample is positive, however, then each one of the separate samples must be tested individually. To be specific, suppose that the blood samples of 10 people are pooled together in this procedure, and suppose that the proportion of those who have the disease in the population from which the samples are drawn is 0.02. (Also assume that the presence of the disease in one sample is independent of the presence of the disease in the other samples.)

a) Find the probability that the pooled sample is negative.

b) Let  $X$  be the number of tests that must be run. Construct the probability distribution of  $X$ .

$x$	$f(x)$
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c) Compute the mean of  $X$ . Is this larger or smaller than 10?

**Example 8:**

Homer Simpson is going to Moe's Tavern for some *Flaming Moe's*. Let  $X$  denote the number of *Flaming Moe's* that Homer Simpson will drink.

Suppose  $X$  has the following probability distribution:

$x$	$f(x)$
0	0.1
1	0.2
2	0.3
3	0.3
4	

a) Find the missing probability  $f(4) = P(X = 4)$ .

b) Find the probability  $P(X \geq 1)$ .

c) Find the probability  $P(X \geq 1 \mid X < 3)$ .

d) Compute the expected value of  $X$ ,  $E(X)$ .

e) Compute the standard deviation of  $X$ ,  $SD(X)$ .

**Example 8: (cont.)**

Suppose each *Flaming Moe* costs \$1.50, and there is a cover charge of \$1.00 at the door. Let  $Y$  denote the amount of money Homer Simpson spends at the bar. Then  $Y = 1.50 \cdot X + 1.00$ .

f) Find the probability that Homer would spend over \$5.00.

g) Find the expected amount of money that Homer Simpson would spend.

h) Find the standard deviation for the amount of money that Homer Simpson would spend