Binomial Distribution

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Binomial Distribution:

1. The number of trials, n, is fixed.
2. Each trial has two possible outcomes: "success" and "failure".
3. The probability of "success", p, is the same from trial to trial.
4. The trials are independent.
5. X = number of "successes" in n independent trials.

Then

$$P(X = k) = \binom{n}{k} \cdot p^{k} \cdot (1-p)^{n-k} = {}_{n}C_{k} \cdot p^{k} \cdot (1-p)^{n-k},$$

where $k = 0, 1, \ldots, n$.

$$E(X) = n \cdot p \qquad Var(X) = n \cdot p \cdot (1-p) \qquad SD(X) = \sqrt{n \cdot p \cdot (1-p)}$$

<u>Example 1</u>:

Bart Simpson takes a multiple-choice exam in his Statistics 101 class. The exam has 15 questions, each has 5 possible answers, only one of which is correct. Bart did not study for the exam, so he guesses independently on every question.
Let X denote the number of questions that Bart gets right.

a) Is it appropriate to use Binomial model for this problem?

b) What is the expected number of questions that Bart would get right?

c) What is the probability that Bart answers exactly 3 questions correctly?

d) What is the probability that Bart would get at most 5 of the questions right?

e) What is the probability that Bart would get more than half of the questions right (i.e. what is the probability that Bart would get at least 8 of the questions right)?

f) Find the probability that Bart answers between 4 and 6 (including both 4 and 6) questions correctly?

R: `pbinom(q, size, prob)` `dbinom(x, size, prob)`

<u>Example 2</u>: ☺

An automobile salesman thinks that the probability of making a sale is 0.30. If he talks to five customers on a particular day, what is the probability that he will make exactly 2 sales? (Assume independence.)