STAT 400 Continuous Random Variables UIUC

Continuous Random Variables

The probabilities associated with a continuous random variable X are determined by the **probability density function** of the random variable. The function, denoted f(x), must satisfy the following properties:

- $f(x) \ge 0$ for all x. 1.
- 2. The total area under the entire curve of f(x) is equal to 1.00.

Then the probability that X will be between two numbers a and b is equal to the area under f(x) between a and b.



For any point c,

P(X = c) = 0.

 $P(a \le X \le b) = P(a \le X < b) = P(a < X \le b) = P(a < X < b).$ Therefore,

Expected value (mean, average):
$$\mu_{\rm X} = \int_{-\infty}^{\infty} x \cdot f(x) \, dx.$$

Variance:

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$$\sigma_X^2 = E\left[(X - \mu_X)^2\right] = \int_{-\infty}^{\infty} (x - \mu_X)^2 \cdot f(x) \, dx.$$

$$\sigma_X^2 = E\left(X^2\right) - [E(X)]^2 = \left[\int_{-\infty}^{\infty} x^2 \cdot f(x) \, dx\right] - (\mu_X)^2.$$
Moment Generating Function:

$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} \cdot f(x) \, dx.$$

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Example 1:

Let X be a continuous random variable with the probability density function



a) What must the value of k be so that f(x) is a probability density function?

b) Find the cumulative distribution function of X, $F_X(x) = P(X \le x)$.

c) Find the probability $P(1 \le X \le 2)$.

d) Find the median of the distribution of X. That is, find *m* such that $P(X \le m) = P(X \ge m) = \frac{1}{2}$.

e) Find the 30th percentile of the distribution of X. That is, find a such that $P(X \le a) = 0.30$.

f) Find $\mu_X = E(X)$.

g) Find $\sigma_X = SD(X)$.

Example 2:

Let X be a continuous random variable with the cumulative distribution function

$$\mathbf{F}(x) = 0, \qquad x < 0,$$

$$F(x) = \frac{3}{8} \cdot x, \qquad 0 \le x \le 2,$$

$$F(x) = 1 - \frac{1}{x^2}, \qquad x > 2.$$



a) Find the probability density function f(x).

b) Find the probability $P(1 \le X \le 4)$.

c) Find $\mu_X = E(X)$.

d) Find $\sigma_X = SD(X)$.