# Uniform and Exponential Distributions

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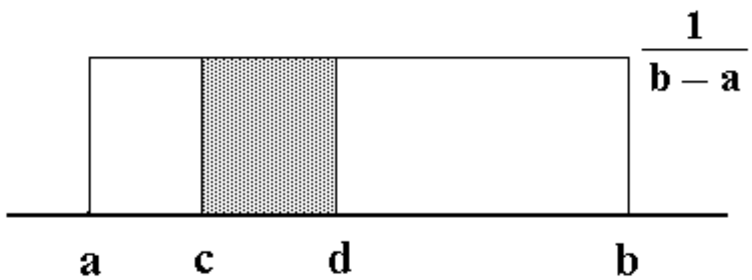
**Uniform Distribution** over an interval [ a , b ]:




For Uniform distribution,

$$P(c \le X \le d) = \frac{d-c}{b-a}, \qquad a \le c \le d \le b.$$

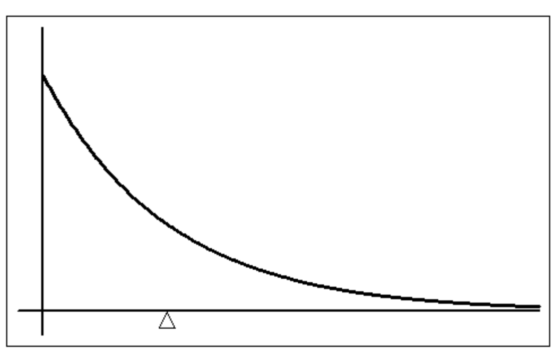
$$E(X) = \frac{a+b}{2}, \qquad Var(X) = \frac{(b-a)^2}{12}.$$

**Exponential Distribution**:

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & \text{for } x \ge 0 \\ 0 & \text{otherwise} \end{cases} \qquad f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

$E(X) = \theta$,

$Var(X) = \theta^2$.



$E(X) = {}^1/_\lambda$,

$Var(X) = {}^1/_{\lambda^2}$.

**<u>Example 1</u>:**

Suppose the lifetime of a particular brand of light bulbs is exponentially distributed with mean of 400 hours.

a) Find the probability that a randomly selected light bulb would last over 500 hours.

b) Find the probability that 3 out of 7 randomly selected light bulbs would last over 500 hours.

c) Find the probability that a randomly selected light bulb would last between 300 hours and 800 hours.

**<u>Example 2</u>:**

Let $X$ be an exponential random variable with mean $\theta$.

$$X \sim Exp(\theta)$$

$$f_X(x) = \frac{1}{\theta} e^{-x/\theta}, \quad x \geq 0.$$

a) Find the formula for $P(X > t)$ for $t > 0$.

b) Show that for positive $t$ and $s$,

$$P(X > t + s \mid X > t) = P(X > s)$$

**Geometric Distribution**: $X \sim Geom(p)$

a) Show that $P(X > a) = (1 - p)^a, \;\; a = 0, 1, 2, \ldots$

b) Show that $P(X > a + b \mid X > a) = P(X > b)$ for $a, b > 0$.