

Example 1: (cont.)

Consider the following joint probability distribution $p(x, y)$ of two random variables X and Y:

$x \setminus y$	0	1	2	
1	0.15	0.10	0	0.25
2	0.25	0.30	0.20	0.75
	0.40	0.40	0.20	

Recall: A and B are independent if and only if $P(A \cap B) = P(A) \cdot P(B)$.

a) Are events $\{X = 1\}$ and $\{Y = 1\}$ independent?

Def Random variables X and Y are **independent** if and only if

discrete $p(x, y) = p_X(x) \cdot p_Y(y)$ for all x, y .

continuous $f(x, y) = f_X(x) \cdot f_Y(y)$ for all x, y .

$$F(x, y) = P(X \leq x, Y \leq y). \quad f(x, y) = \partial^2 F(x, y) / \partial x \partial y.$$

Def Random variables X and Y are **independent** if and only if

$$F(x, y) = F_X(x) \cdot F_Y(y) \quad \text{for all } x, y.$$

b) Are random variables X and Y independent?

Example 2:

Let the joint probability density function for (X, Y) be

$$f(x, y) = \begin{cases} 60x^2y & 0 \leq x \leq 1, 0 \leq y \leq 1, x + y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Recall: $f_X(x) = 30x^2(1-x)^2, \quad 0 < x < 1,$

$$f_Y(y) = 20y(1-y)^3, \quad 0 < y < 1.$$

Are random variables X and Y independent?

Example 3:

Let the joint probability density function for (X, Y) be

$$f(x, y) = \begin{cases} x + y & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Are X and Y independent?

Example 4:

Let the joint probability density function for (X, Y) be

$$f(x, y) = \begin{cases} 12x(1-x)e^{-2y} & 0 \leq x \leq 1, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Are X and Y independent?

If random variables X and Y are independent, then

$$E(g(X) \cdot h(Y)) = E(g(X)) \cdot E(h(Y)).$$

Example 5:

Suppose the probability density functions of T_1 and T_2 are

$$f_{T_1}(x) = \alpha e^{-\alpha x}, \quad x > 0, \quad f_{T_2}(y) = \beta e^{-\beta y}, \quad y > 0,$$

respectively. Suppose T_1 and T_2 are independent. Find $P(2T_1 > T_2)$.

Example 6:

Let X and Y be two independent random variables, X has a Geometric distribution with the probability of “success” $p = 1/3$, Y has a Poisson distribution with mean 3. That is,

$$p_X(x) = \left(\frac{1}{3}\right) \cdot \left(\frac{2}{3}\right)^{x-1}, \quad x = 1, 2, 3, \dots,$$

$$p_Y(y) = \frac{3^y e^{-3}}{y!}, \quad y = 0, 1, 2, 3, \dots$$

a) Find $P(X = Y)$.

b) Find $P(X = 2Y)$.

a) Find $P(X > Y)$.