

Confidence Intervals for Means

Sample Size Planning

Example 1:

Suppose the lifetime of a particular brand of light bulbs is normally distributed with standard deviation of $\sigma = 75$ hours and unknown mean.

a) What is the probability that in a random sample of $n = 49$ bulbs, the average lifetime \bar{X} is within 21 hours of the overall average lifetime?

b) Suppose the sample average lifetime of $n = 49$ bulbs is $\bar{x} = 843$ hours. Construct a 95% confidence interval for the overall average lifetime for light bulbs of this brand.

A **confidence interval** is a *range of numbers* believed to include an unknown population parameter. Associated with the interval is a measure of the *confidence* we have that the interval does indeed contain the parameter of interest.

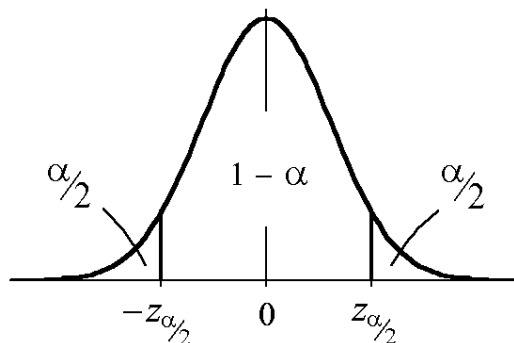
A $(1 - \alpha)$ 100% confidence interval

for the population mean μ

when σ is known

and sampling is done from a normal population, or with a large sample, is

$$\left(\bar{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \right)$$



\bar{X}	\pm	$z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$	$\bar{X} \pm \varepsilon$
estimate (point estimate)		margin of error ε	$\varepsilon = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

Example 1: (continued)

b) Suppose the sample average lifetime of $n = 49$ bulbs is $\bar{x} = 843$ hours. Construct a 95% confidence interval for the overall average lifetime for light bulbs of this brand.

c) Construct a 90% confidence interval for the overall average lifetime for light bulbs.

d) Construct a 92% confidence interval for the overall average lifetime for light bulbs.

Minimum required sample size in estimating the population mean μ to within ϵ with $(1 - \alpha) 100\%$ confidence is

$$n = \left[\frac{Z_{\alpha/2} \cdot \sigma}{\epsilon} \right]^2 .$$

Always round n up.

Example 2:

How many test runs of an automobile are required for determining its average miles-per-gallon rating on the highway to within 0.5 miles per gallon with 95% confidence, if a guess is that the variance of the population of miles per gallon is about 6.25?

Example 1: (continued)

e) What is the minimum sample size required if we wish to estimate the overall average lifetime for light bulbs to within 10 hours with 90% confidence?