Confidence Intervals for Variances and Standard Deviations

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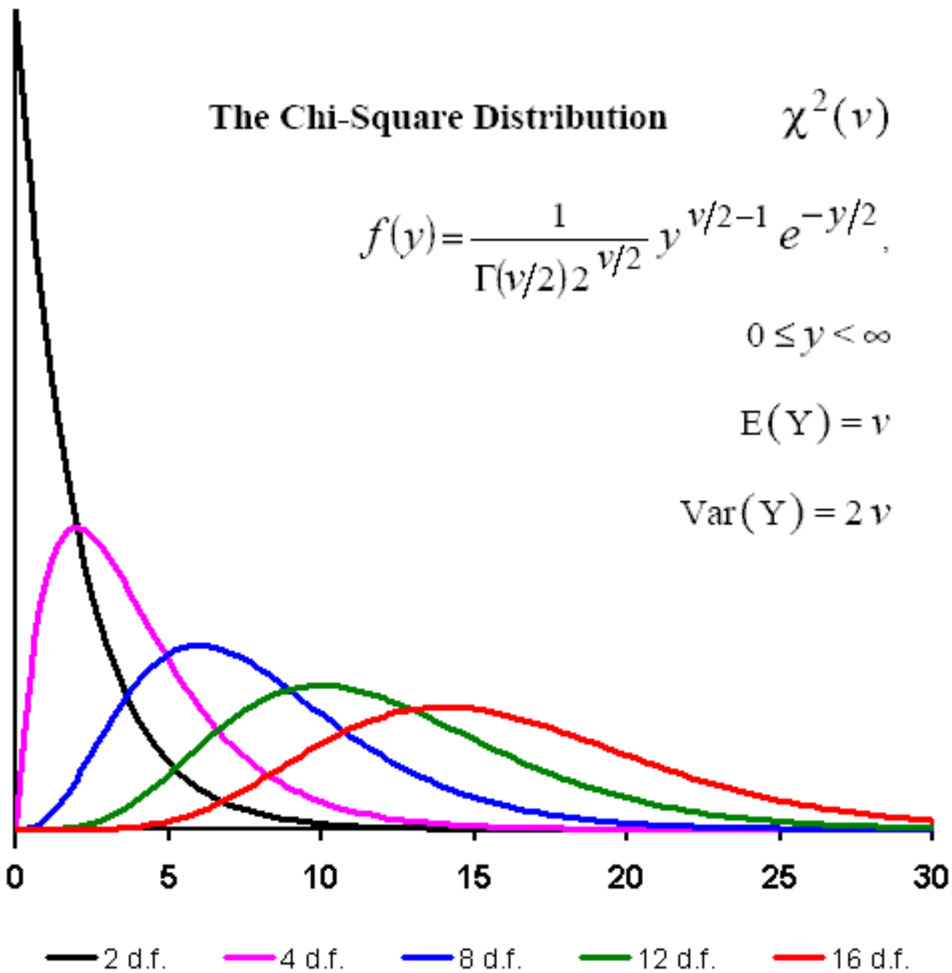


Recall:

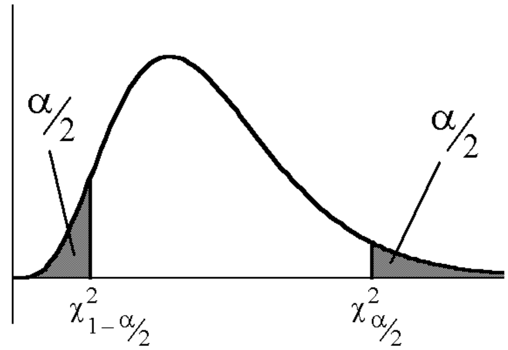
If $X_1, X_2, \ldots, X_n$ are i.i.d. $\mathbf{N}(\mu, \sigma^2)$. Then

$$\frac{(n-1)\cdot S^2}{\sigma^2} = \frac{\sum (X_i - \overline{X})^2}{\sigma^2} \text{ is } \chi^2(n-1).$$

A $(1-\alpha)\,100\%$ confidence interval for the population variance σ^2
(where the population is assumed normal)

$$\left(\frac{(n-1)\cdot s^2}{\chi^2_{\alpha/2}},\ \frac{(n-1)\cdot s^2}{\chi^2_{1-\alpha/2}} \right)$$

$n-1$ degrees of freedom



A $(1-\alpha)\,100\%$ confidence interval for the population standard deviation σ (where the population is assumed normal)

$$\left(\sqrt{\frac{(n-1)\cdot s^2}{\chi^2_{\alpha/2}}},\ \sqrt{\frac{(n-1)\cdot s^2}{\chi^2_{1-\alpha/2}}} \right) \quad \text{OR} \quad \left(s\cdot\sqrt{\frac{(n-1)}{\chi^2_{\alpha/2}}},\ s\cdot\sqrt{\frac{(n-1)}{\chi^2_{1-\alpha/2}}} \right)$$

$n-1$ degrees of freedom

Example 1:
A machine makes ½-inch ball bearings. In a random sample of 41 bearings, the sample standard deviation of the diameters of the bearings was 0.02 inch. Assume that the diameters of the bearings are approximately normally distributed. Construct a 90% confidence interval for the standard deviation of the diameters of the bearings.

Example 2:

The following random sample was obtained from $N(\mu, \sigma^2)$ distribution:

16	12	18	13	21	15	8	17

Recall: $\bar{x} = 15$, $s^2 = 16$, $s = 4$.

a) Construct a 95% confidence interval for the overall standard deviation.

b) Construct a 95% confidence lower bound for the overall standard deviation.

c) Construct a 95% confidence upper bound for the overall standard deviation.